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CUBIC STRUCTURAL UNIT OF THE CLASS THETA=1 TENSEGRITY SYSTEM

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ABSTRACT

The manner of material layout is essential to strength at all scales, from nanoscale biological systems to megascale civil structures. Beyond doubt, in a high degree impractical is to explain the structural strength of object via continuum of material. Generally, we need a fundamental knowledge, which involve the geometrical analysis of shapes and the relationships among them. The geometry of those shapes determines their functionality. Of all the engineering disciplines, probably civil engineering uses geometry the most.

Civil structures for ages tended to be made with orthogonal beams, columns, and plates. The knowledge and experience suggest that this gravity-based architecture does not usually yield the minimal mass design for a given collection of criteria.

It is a general knowledge that material is needed only in the fundamental load paths, not the orthogonal directions of traditional manmade structures. Moreover, for majority materials, the tensile strength of a longitudinal member is larger than its buckling strength.

The emergence of tensegrity systems opens new fields for conceptual design[1,2,4]. The investigation process of tensegrity can be approximately divided into four steps: finding of topological scheme, self-equilibrium analysis, stability analysis, and mechanical reaction analysis.

According to pioneer idea, tensegrity structures are free-standing and prestressed pin-jointed structures, which are different from other types of structures, such as trusses carrying no prestress or cable-nets attached to supports. Configurations of the structures carrying prestresses cannot

be arbitrarily determined, because the nodes and members have to be in the balance of prestresses. Hence, form-finding is a basic and important problem for design of tensegrity structures. As the first step, topology design focuses on the possible connections between cables and bars to satisfy the tensegrity principle.

According to the initially accepted definition, a tensegrity structure is a prestressed pin-jointed structure consisting of discontinuous bars and continuous cables.

Robert Skelton proposed a more scientific definition: “A tensegrity system is a stable connection of axially-loaded members. A *Class* $\boldsymbol{k}$ tensegrity structure is one in which at most $\boldsymbol{k}$ compressive members are connected to any node”[3]. The reservation of a continuous network of cables in tension has been implicitly preserved.

Starting from the above definition of a Class $\boldsymbol{k}$ tensegrity structure, it should be stated, that among tensegrity systems there are also examples with a discontinuous network of cables[4-6].

It is possible to design a separate set of cables inside the cable-bar elementary cell and to establish a self-stress state of equilibrium. Each of the basic tensegrity systems termed *Class Theta*, or *Class* $\boldsymbol{\Theta}$ by means of symbol, possesses an external and internal set of tension components. The shape of Greek capital letter $\boldsymbol{\Theta}$ (Theta) reflects two separate sets of such components (two sets of tendons, cables etc.).

Thus, it should be understood as follows; both $\boldsymbol{k}=1$ and $\boldsymbol{\Theta}=1$ means that the proper set of cables is joined at most to one bar in each node. Moreover no bars are in contact each other. The results of research on the class $\boldsymbol{\Theta}=1$ tetrahedron and triangular prism are presented in [4-12].

It is also known that besides $\boldsymbol{k}>1$ systems in autonomous equilibrium there are tensegrity systems $\boldsymbol{\Theta}>1$[13-15].

Currently, other physical, geometrical and mathematical model of the $\boldsymbol{\Theta}=1$ class tensegrity system is presented. It has a unique external cubic form in comparison with all tensegrity systems known so far, as shown in Fig.1. For obvious reasons, this basic form makes it very easy to shape, for example, modular engineering structures.

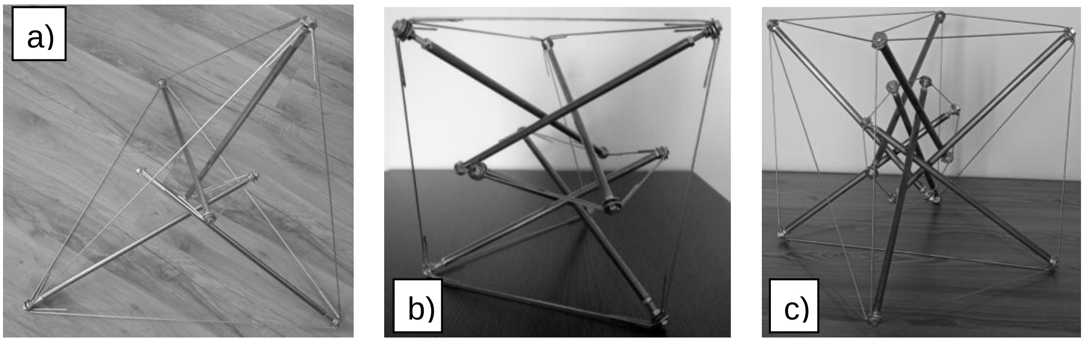

Fig. 1 Examples of the Class $\boldsymbol{\Theta}=1$ tensegrity systems with separable bars, i.e. “pure” tensegrity systems: (a) tetrahedron; (b) triangular prism; (c) cube.

Due to the unique mechanical characteristics; both with and without loads, *Class* ***Θ*** tensegrity structures can hold various applications in the design of civil architectures, advanced/architected materials, smart devices, biomechanical models and many others.

Let's take into account that progress in manufacturing technologies already allows the production of architected materials, also known as metamaterials, with so far unprecedented properties. Most such materials are characterized by a fixed geometry, but in the design of some materials it is possible to incorporate internal mechanisms capable of reconfiguring their spatial architecture, and in this way to enable tunable functionality.

Figure 2 shows the geometric model of a tensegrity cube of class ***Θ*** =1, in which the length of all bars is identical to the length of external cables of system.

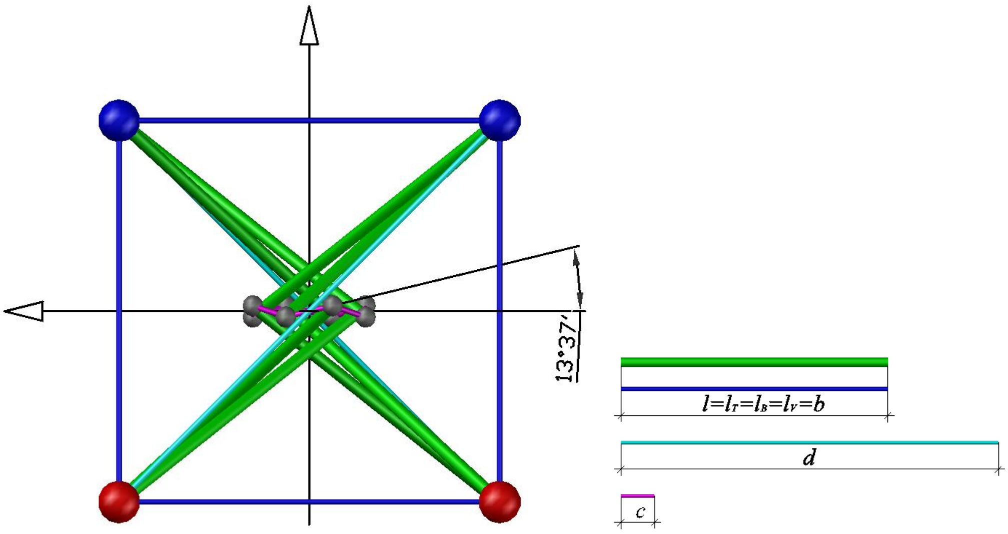

Fig 2. For *l*=100 and *b*=100 the angle *α* equals 13°37', and the length of internal cables *c*=12.613658

The following is a mathematical model for figures related to the Class ***Θ***=1 cubic structural unit, explaining why the tensegrity module is a stable construction, albeit with infinitesimal mobility.

Consider a cube centred at origin of Cartesian system with edges length *l*. The length *b* of the bars can be easily computed by using the following espression:

$$b = \left(\left(\frac{1+\sqrt{2}}{2}\right)^2 \cdot c^2 \cdot \cos^2\alpha + \left(1+\frac{\sqrt{2}}{2}\right) \cdot c \cdot l \cdot \cos\alpha + \frac{1}{2} c \cdot l \cdot \sin\alpha + \frac{c^2+3\,l^2}{4}\right)^{\frac{1}{2}} \qquad (1)$$

By varying the values of a and b and the lengths of the connectivities, we could design different configurations. Since the tensegrity represents an extremal point of the relation recorded in (1), it has infinitesimal mobility.

Figure 3 exhibits the fragmentary effect of interpretation (1) in the Matlab program for *l*=*b*=100 and *c* = 0.05.

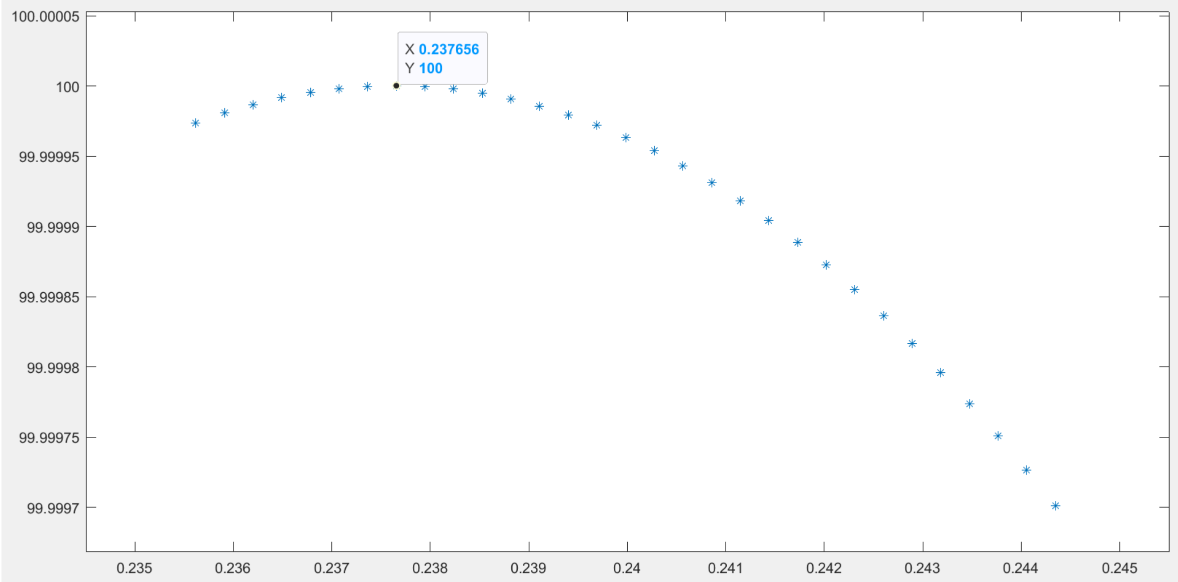

Fig. 3 The Matlab graph $b_{max} = f(c,\alpha)$

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