Committee for Architecture and Town Planning
Wroclaw Branch of the Polish Academy of Sciences
International Scientific Workshops „Innovative Structural Systems in Architecture”
(ISSA2023) November, 9-10, 2023, Wroclaw, Poland

CONSTANT STRESS PRINCIPLE VS MINIMUM WEIGHT IN DESIGN OF ARCH STRUCTURES.

Wanda LEWIS*

* School of Engineering
University of Warwick
Coventry CV4 7AL, UK

E-mail: w.j.lewis@warwick.ac.uk URL: https://warwick.ac.uk/fac/sci/eng/people/wjl

Keywords: *constant stress arch, optimal arch form, analytical form-finding, minimum weight principle*

ABSTRACT

Introduction

The term ‘optimal’ is much used and abused in the field of structural design. An ‘optimal’ structure is understood to have a minimum weight necessary to resist an ultimate load. However, it can be shown that these features do not necessarily produce an ‘optimal’ design solution. Minimum weight is a criterion commonly adopted in structural optimisation, and it is also an underlying principle in the current Limit State Design philosophy. My latest work on analytical form-finding of moment-less arches [1] shows that this criterion can limit design options, and when used in conjunction with the ultimate load, affect durability. The arches under consideration are two-pin structures that do not experience bending action under statistically prevalent load, such as structure’s self-weight. Thus, they work in their optimum state of constant stress most of the time, unlike structures shaped by an ultimate load, which makes them optimal only for this transient load. Of course, a check on stresses under the ultimate load is necessary and, in the event of ultimate stresses exceeding the design strength of material, a lower value of constant stress can be used, as explained in [2].
The analytical form-finding methodology is capable of producing both symmetric and asymmetric arch forms. Figure 1(a) and (b) gives examples of symmetric forms of constant stress arches. The input data indicates that the geometry of the arch is a function of: span, rise, arch and deck loading, and a value of constant stress. The output is given in the form of reactions, centre-line profile, and a varying cross-section area. The cross-section can

be either solid or hollow, and can adopt any shape, depending on the type of material used, stability requirements, and aesthetic quality.

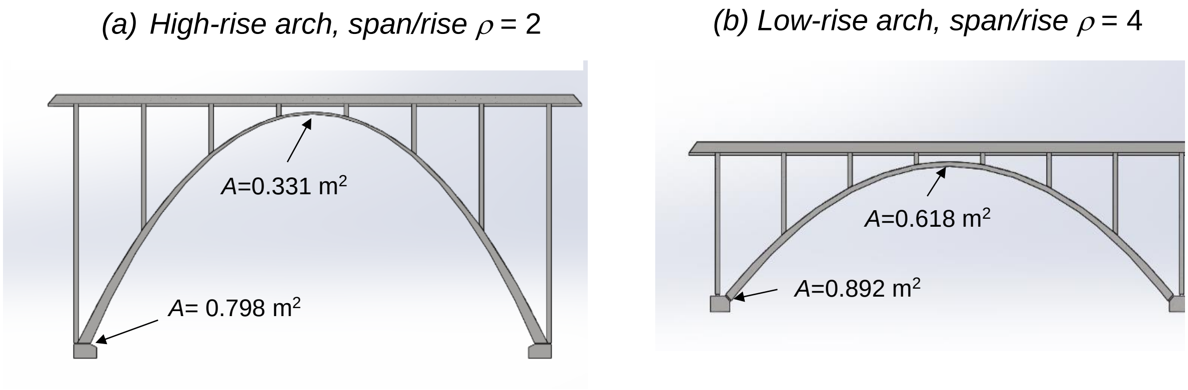

Fig. 1. Models of symmetric 2-pin arches of constant axial stress. Input: span = 50 m, arch weight density = 25 kN/m^3, deck load = 50 kN/m, constant stress = 2.4 MPa in (a), and 2.75 MPa in (b). Output: centre-line profile, varying cross-section area, A, and reactions [2]. CAD models: M. Millson, University of Warwick

It can be seen that the variation of the cross-section area in the two arches is greater in the case of the high-rise arch (Fig.1 (a)) than in the low-rise one (Fig.1 (b)). This reflects the variation of the axial force in the two structures.

Figure 2 shows an asymmetric form, proposed as a replacement of an existing, 1965 bridge, at the University of Warwick (Fig. 3).

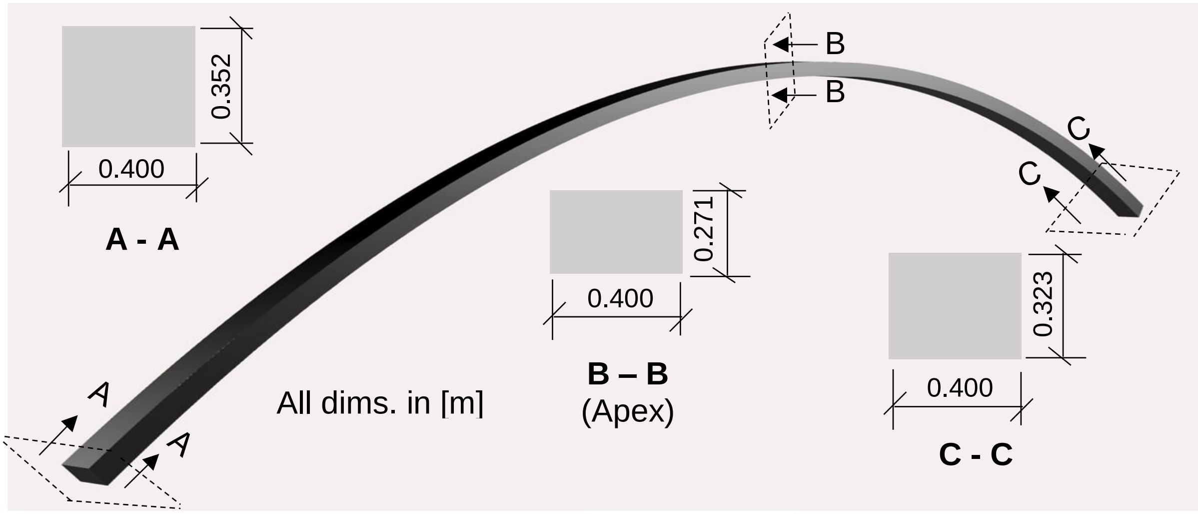

Fig. 2. 3D view of a model of an asymmetric constant stress arch, scale 1:50. Input: span = 23.7 m, span/rise, ρ = 3.7, difference in height at supports = 1.929 m, Arch weight density 25 kN/m^3, deck load = 11.75 kN/m, constant stress 1.8 MPa. Output: distance to the apex from left support = 13.171 m, cross-section areas: $A_{B\text{-}B}$ = 0.1084 m^2, $A_{A\text{-}A}$ = 0.1408m^2, $A_{C\text{-}C}$ = 0.1292m^2. CAD model: M. Millson, University of Warwick

Figure 3 shows an in-situ visualisation of the asymmetric arch presented in Fig. 2.

(a)

(b)

Fig. 3. (a) Proposed constant stress arch (model shown in Fig. 2) as a replacement of the existing 1965 ‘beam and column’ design shown in Fig. 3(b).

Constant stress arches are moment-less but, unlike moment-less aches of constant cross-section discussed in [3], their cross-section area varies to ensure that the axial compressive stress does not change along the length of the arch. Advantages of both types of moment-less arches over dictated forms of parabolic or circular configurations are discussed in [2] and [3].

Common with other natural structural forms, such as minimal surface roofs modelled on the principle of constant surface stress [4], constant stress arches cannot be formed for any set of input parameters; their existence is determined by a combination of them, determining their Design Space – a concept discussed in [1]. Although it may be viewed as a limitation, the Design Space allows ample opportunities for generating numerous constant stress arch forms.

Figure 4 examines the effect of the main input parameter, such as the constant stress value, *f*, on the volume of material used in the arch, for different values of span/rise ratios, ρ. The data concerns the arch shown in Fig. 1(a).

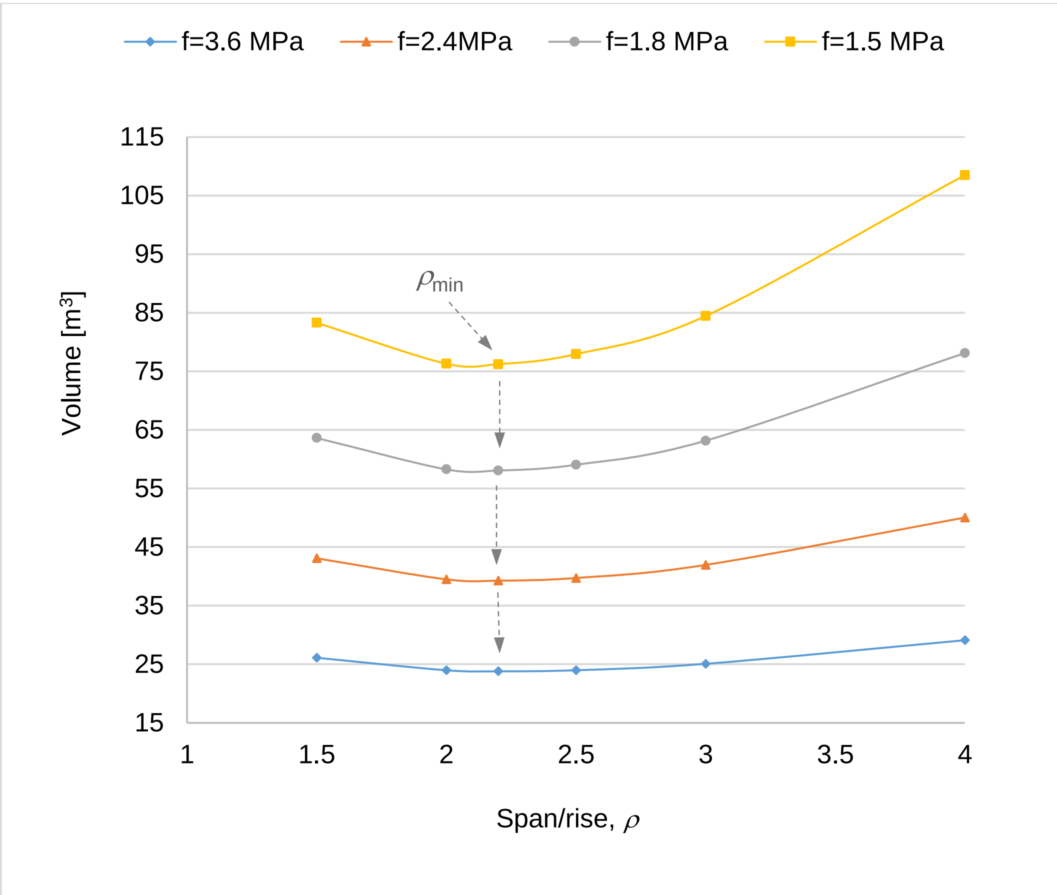

Fig. 4. Volume of material in the constant stress arch shown in Fig. 1, as a function of varying span/rise ratios, ρ, (including the volume-minimising ratio, ρ_{min}), and different values of chosen constant stress, *f*.

It can be seen that, for the given input parameters, the volume-minimising ratio, ρ_{min}, for the arch is around 2.2. It can also be seen that constant stress arches are not necessarily structures of minimum weight.

It is a common misconception that minimum weight structures are 'optimal'. In reality, they produce a volume-minimising span/rise ratio that may, or may not, suit a particular landscape of a building site, required headroom, etc. Satisfying this ratio may require additional building and groundworks, increasing project costs. As stated earlier, when modelled for the ultimate load, minimum weight structures are optimal only for this transient load state. In contrast, constant stress arches give the designer a choice of span/rise ratios, while still providing solutions in the form of structures that have a minimal stress response to statistically prevalent loading - a feature observed in natural objects, such as shells, bones, trees [5]. This characteristic puts constant stress arches into a category of biomimetic structures. The arch bridge shown in Fig. 3(a) is currently going through approval stages at the University of Warwick; when built, it will be the first structure of this type in the world.

The feature of predominantly constant stress under statistically prevalent loading contributes to improved durability, giving these arches the potential to address sustainability issues facing our future infrastructure.

Acknowledgement

This work has been supported by the Leverhulme Trust [grant ref: EM-2020-059/9].

REFERENCES

[1] W. J. Lewis: *Constant stress arches and their design space*, Proc. R. Soc. A, 2021, 78 (doi: 10.1098/rspa. 2021.0428).

[2] W. J. Lewis, J. Russell, and T. Q. Li, *Moment-less arches of constant axial stress. Implications for design,* Engineering Structures, https://doi.org/10.1016/j.engstruct.2023.116968 2023.

[3] W. J. Lewis, J. Russell, and T. Q. Li, *Moment-less arches for reduced stress state*. *Comparisons with conventional arch forms,* Computers and Structures, 2021, 251 (doi.org/10.1016/j.compstruc.2021.106524).

[4] F. Otto and B. Rasch, *Finding form. Towards an architecture of the minimal*. Deutscher Werkbund Bayern. Edition Axel Menges, 1995.

[5] C. Mattheck C. *Design in nature. Learning from trees*. Berlin Heidelberg: Springer 1998.